PROBLEMA 1

$$L(x,\theta) = \theta^n (1-\theta)^{\sum x_i - n}$$

$$\ln L(x,\theta) = n \ln \theta + (\sum x_i - n) \ln (1-\theta)$$

C.P.O. $\dfrac{\partial \ln L(x,\theta)}{\partial \theta} = \dfrac{n}{\theta} + \dfrac{\sum x_i - n}{1-\theta}(-1) = 0 \qquad \dfrac{n}{\theta} - \dfrac{\sum x_i - n}{1-\theta} = 0 \Rightarrow$

$$\frac{n(1-\theta) - (\sum x_i - n)\theta}{\theta(1-\theta)} = 0 \Rightarrow n - \cancel{n\theta} - \theta \sum x_i + \cancel{n\theta} = 0 \Rightarrow n = \theta \sum x_i \Rightarrow$$

$$\hat{\theta}_{MV} = \frac{n}{\sum x_i} = \frac{1}{\overline{x}}$$

PROBLEMA 2

Una sola incógnita (λ), por tanto una sola ecuación : $\overline{x} = \mu$

$$\mu = \int_0^\lambda x \frac{2(\lambda - x)}{\lambda^2} dx = \frac{\lambda}{3}$$

Por tanto $\overline{x} = \dfrac{\lambda}{3} \Rightarrow \hat{\lambda}_{MOM.} = 3\overline{x}$

PROBLEMA 3

$B(m,p)$ donde $f(x,p) = \binom{m}{x} p^x (1-p)^{m-x}$

(A) $L(x,p) = \prod \binom{m}{x_i} p^{\sum x_i} (1-p)^{m \cdot n - \sum x_i}$

$$\ln L(x,p) = \ln\left(\prod \binom{m}{x_i}\right) + \sum x_i \ln p + (m \cdot n - \sum x_i) \ln (1-p)$$

$$\frac{\partial \ln L(x,p)}{\partial p} = \frac{\sum x_i}{p} - \frac{m \cdot n - \sum x_i}{1-p} = 0 \Rightarrow \hat{p}_{MV} = \frac{\overline{x}}{m}$$

(B) Una sola incógnita (p), por tanto una sola ecuación $\overline{x} = \mu$

donde $\mu = m \cdot p$

Por tanto $\overline{x} = m \cdot p \Rightarrow \hat{p}_{MOM} = \dfrac{\overline{x}}{m}$

PROBLEMA 4

(A) $L(x,\theta) = \left(\dfrac{2\theta}{1-\theta}\right)^n \prod x_i^{\frac{3\theta-1}{1-\theta}}$

$$\ln L(x,\theta) = n \ln\left(\frac{2\theta}{1-\theta}\right) + \frac{3\theta-1}{1-\theta} \sum \ln x_i =$$

$$= n\ln 2 + n\ln\theta - n\ln(1-\theta) + \frac{3\theta-1}{1-\theta}\sum \ln x_i$$

$$\frac{\partial \ln L(x,\theta)}{\partial\theta} = \frac{n}{\theta} + \frac{n}{1-\theta} + \frac{2}{(1-\theta)^2}\sum \ln x_i = 0$$

$$\Rightarrow \hat{\theta}_{MV} = \frac{n}{n - 2\sum \ln x_i}$$

(B) Una sola incógnita (θ), por tanto una sola ecuación: $\overline{x} = \mu$

$$\mu = \int_0^1 x \cdot \frac{2\theta}{1-\theta} x^{\frac{3\theta-1}{1-\theta}} dx = \frac{2\theta}{1+\theta}$$

Por tanto $\overline{x} = \dfrac{2\theta}{1+\theta} \Rightarrow \hat{\theta}_{MOM} = \dfrac{\overline{x}}{2-\overline{x}}$