PROBLEMA 1

$\odot$ $E(A) = E\left[\frac{\Sigma x_i}{2n}\right] = \frac{1}{2n} E(\Sigma x_i) = \frac{n\mu}{2n} = \frac{\mu}{2}$

pero $\mu = \int_0^\infty x \cdot \frac{x}{a^2} \cdot e^{-\frac{x}{a}} dx = 2a$, por tanto

$E(A) = \frac{\mu}{2} = \frac{2a}{2} = a \Rightarrow$ INSESGADO.

$\odot$ $V(A) = V\left[\frac{\Sigma x_i}{2n}\right] = \frac{1}{4n^2} V(\Sigma x_i) = \frac{n\sigma^2}{4n^2} = \frac{\sigma^2}{4n}$

pero $\sigma^2 = \alpha_2 - \mu^2$ donde $\alpha_2 = \int_0^\infty x^2 \cdot \frac{x}{a^2} e^{-\frac{x}{a}} dx = 6a^2$, por tanto $\sigma^2 = 6a^2 - (2a)^2 = 2a^2$

$V(A) = \frac{2a^2}{4n} = \frac{a^2}{2n}$

Cota de Cramer-Rao para estimadores insesgados:

$$\frac{1}{nE\left[\frac{\partial \ln f(x,a)}{\partial a}\right]^2}$$

$\ln f(x,a) = \ln x - 2\ln a - \frac{x}{a}$

$\frac{\partial \ln f(x,a)}{\partial a} = -\frac{2}{a} + \frac{x}{a^2} = \frac{x-2a}{a^2}$

$E\left[\frac{x-2a}{a^2}\right]^2 = \frac{1}{a^4} \cdot \underbrace{E(x-2a)^2}_{\sigma^2 = 2a^2} = \frac{2a^2}{a^4} = \frac{2}{a^2}$

Cota de C-R $= \frac{1}{n \cdot \frac{2}{a^2}} = \frac{a^2}{2n}$

$V(A) =$ Cota C-R $\Rightarrow$ A es EFICIENTE

PROBLEMA 2

$\odot\, E(S_1^2) = \sigma^2 \Rightarrow$ INSESGADO

$\odot\, V(S_1^2) = \dfrac{2\sigma^4}{n-1} \xrightarrow[n\to\infty]{} 0$

$\Rightarrow$ CONSISTENTE

PROBLEMA 3

$$\odot\, E(\hat{p}) = E\left[\frac{1}{5}\bar{x}\right] = \frac{1}{5}E(\bar{x}) = \frac{1}{5}\cdot\mu = \frac{1}{\cancel{5}}\cancel{5}p = p \Rightarrow \text{INSESGADO}$$

$\mu = 5p$

$$\odot\, V(\hat{p}) = V\left(\frac{1}{5}\bar{x}\right) = \frac{1}{5^2}V(\bar{x}) = \frac{1}{5^2}\frac{\sigma^2}{n} = \frac{1}{5^{\cancel{2}}}\frac{\cancel{5}p(1-p)}{n} = \frac{p(1-p)}{5n}$$

$\sigma^2 = 5p(1-p)$

Cota de Cramer-Rao para estimadores insesgados:

$$\frac{1}{nE\left[\frac{\partial \ln f(x,p)}{\partial p}\right]^2}$$

$$f(x,p) = \binom{5}{x}p^x(1-p)^{5-x}$$

$$\ln f(x,p) = \ln\binom{5}{x} + x\ln p + (5-x)\ln(1-p)$$

$$\frac{\partial \ln f(x,p)}{\partial p} = \frac{x}{p} - \frac{5-x}{1-p} = \frac{x-5p}{p(1-p)}$$

$$E\left[\frac{x-5p}{p(1-p)}\right]^2 = \frac{1}{p^2(1-p)^2}\underbrace{E\left[x-5p\right]^2}_{\sigma^2=5p(1-p)} = \frac{5\cancel{p(1-p)}}{p^{\cancel{2}}(1-p)^{\cancel{2}}} = \frac{5}{p(1-p)}$$

$$\text{Cota de C-R} = \frac{1}{n\cdot\frac{5}{p(1-p)}} = \frac{p(1-p)}{5n}$$

$V(\hat{p}) =$ Cota C-R $\Rightarrow \hat{p}$ es EFICIENTE

PROBLEMA 4

- INSESGADEZ?

$$E(\bar{x})=\mu \quad , \text{donde } \mu=\int_0^\infty x\cdot\frac{1}{\theta}e^{-\frac{1}{\theta}x}dx=\theta$$

por tanto $E(\bar{x})=\theta \Rightarrow$ INSESGADO

- EFICIENCIA?

$$V(\bar{x})=\frac{\sigma^2}{n} \quad , \text{donde } \sigma^2=\alpha_2-\mu^2 \quad , \alpha_2=\int_0^\infty x^2\frac{1}{\theta}e^{-\frac{1}{\theta}x}dx=2\theta^2$$

$$\sigma^2=2\theta^2-\theta^2=\theta^2$$

$$V(\bar{x})=\frac{\theta^2}{n}$$

Cota de Cramer-Rao para estimadores insesgados:

$$\frac{1}{nE\left[\frac{\partial \ln f(x,\theta)}{\partial\theta}\right]^2}$$

$$f(x,\theta)=\frac{1}{\theta}e^{-\frac{1}{\theta}x}$$

$$\ln f(x,\theta)=-\ln\theta-\frac{1}{\theta}x$$

$$\frac{\partial \ln f(x,\theta)}{\partial\theta}=-\frac{1}{\theta}+\frac{x}{\theta^2}=\frac{x-\theta}{\theta^2}$$

$$E\left[\frac{x-\theta}{\theta^2}\right]^2=\frac{1}{\theta^4}\underbrace{E(x-\theta)^2}_{\sigma^2=\theta^2}=\frac{\theta^2}{\theta^4}=\frac{1}{\theta^2}$$

$$\text{Cota de C-R}=\frac{1}{n\cdot\frac{1}{\theta^2}}=\frac{\theta^2}{n}$$

$V(\bar{x})=$ Cota de C-R $\Rightarrow \bar{x}$ es EFICIENTE

(c) CONSISTENCIA?

$$\left.\begin{array}{l}\bar{x} \text{ es insesgado} \\ V(\bar{x}) = \dfrac{\theta^2}{n} \xrightarrow[n\to\infty]{} 0\end{array}\right\} \bar{x} \text{ es CONSISTENTE.}$$

PROBLEMA 5

$$\text{(A)} \quad \left.\begin{array}{l}\bar{x} \text{ es insesgado} \\ V(\bar{x}) = \dfrac{\sigma^2}{n} \xrightarrow[n\to\infty]{} 0\end{array}\right\} \bar{x} \text{ es CONSISTENTE}$$

$$\text{(B)} \quad \left.\begin{array}{l}E(S^2) = \sigma^2 - \dfrac{\sigma^2}{n} \Rightarrow -\dfrac{\sigma^2}{n} \xrightarrow[n\to\infty]{} 0 \Rightarrow \text{ASINTÓTICAMENTE INSESGADO} \\ V(S^2) = \dfrac{2(n-1)\sigma^4}{n^2} \xrightarrow[n\to\infty]{} 0\end{array}\right\} S^2 \text{ es CONSISTENTE}$$

PROBLEMA 6

La funcion de probabilidad conjunta de la muestra (o funcion de verosimilitud) es:

$$L(x, \mu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x_1-\mu)^2} \cdot \ldots\ldots \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x_n-\mu)^2} =$$

$$= \left[\sigma\sqrt{2\pi}\right]^{-n} e^{-\frac{1}{2\sigma^2}\sum(x_i-\mu)^2}$$

Desarrollamos el exponente de e:

$$\sum(x_i-\mu)^2 = \sum x_i^2 + n\mu^2 - 2\mu \underbrace{\sum x_i}_{n\bar{x}} = \sum x_i^2 + n\mu^2 - 2\mu n\bar{x} =$$

$$\sum x_i^2 + n(\mu^2 - 2\mu\bar{x})$$

Definimos las funciones:

$$g(\bar{x};\mu) = e^{-\frac{1}{2\sigma^2}\left[n(\mu^2 - 2\mu\bar{x})\right]}$$

y $$H(x) = \left[\sigma\sqrt{2\pi}\right]^{-n} e^{-\frac{1}{2\sigma^2}\sum x_i^2}$$

Con estas funciones se obtiene la descomposición de la función de verosimilitud en la forma:

$$L(x,\mu) = g(\bar{x};\mu)\cdot H(x)$$

donde $x = (x_1, \ldots, x_n)$

Por tanto $\bar{x}$ es un estimador suficiente.